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Designing parameter optimization of a Parallel-Plain Fin heat sink using the grey-based fuzzy algorithm with the orthogonal arrays

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ABSTRACT

In this paper, a grey-based fuzzy algorithm with the orthogonal arrays is employed to find the optimal designing parameters' setting for a heat sink with the Parallel-Plain Fin (PPF) on the multiple thermal performance characteristics. The proposed algorithm, coupling the grey relational analysis with the fuzzy logic, obtains a grey–fuzzy reasoning grade to evaluate the multiple performance characteristics according to the grey relational coefficient of each performance characteristic. In the present study, the designing parameters of the heat sink include the height of fin, the width of gap between fins, the width of slot, the number of slot and the air speed. The design of experiment (DOE) adopts the $L_{16}(4^5)$ orthogonal arrays table which is four levels and five factors type of factorial design. The average convective heat transfer coefficient, the thermal resistance and the pressure drop are considered as the multiple thermal performance characteristics and explored in the experiment. In addition, the response table, response graph and the analysis of variance (ANOVA) are used to find the optimal settings and the influence of designing parameters on the multiple performance characteristics. The results of confirmation tests with the optimal settings of designing parameters have obviously shown that the multiple thermal performance characteristics are effectively improved through these procedures.

1. Introduction

As the circuit density and power dissipation of integrated circuit chips were upgraded, the heat flux levels within these chips have been relatively increased. The accumulation of a large amount of heat flux would create a considerable quantity of thermal stress on the chips, substrate, and the package. Therefore, it is necessary that the effective cooling device and cooling method are employed in order to maintain the operating temperature of electronic components at a satisfactory level. Due to the enhancement of cooling device in the current electronic industry, the heat sink module is extensively used to provide the cooling capability for electronic components. Generally speaking, the advantage to use the heat sink is its structural simplicity and fair cost. If there is an appropriate and effective design of high-performance cooling capability for the heat sink, it will greatly affect the reliability and life span of chip function. There have been many investigations about the optimum design parameters and the selection of heat sink with a high-performance heat removal characteristic [1–7]. Vollaro *et al.* [1] proposed the optimum design of vertical rectangular fin arrays. Culham and Muzychka [2] utilized the entropy generation minimization for the optimization of plate-fin heat sinks. Iyengar and Bar-Cohen [3] determined the least-energy optimization of plate-fin heat sinks in the status of forced convection. Park *et al.* [4,5] performed an investigation of numerical shape optimization for high performance of a heat sink with pin-fins. Park and Moon [6] proposed the progressive quadratic response surface model to obtain the optimal values of design variables for a plate-fin type heat sink. Yakut *et al.* [7] investigated the effect of design parameters on the heat transfer and pressure drop characteristics of a heat exchanger using the Taguchi experimental design method. From the above descriptive analysis, the optimal design and selection of effective heat sink module become one of the primary challenges in the cooling enhancement of current electronic industry.

The grey relational system based on the grey system theory [8,9] had been proven to be useful for dealing with a poor, incomplete and uncertain information so as to gain the relationship of data among these information. Under the condition having only partial known information, it is desirable for handling an uncertain systematic problem to make use of grey relational generation and calculate the grey relational coefficient. The grey relational coefficient can express the relationship between an objective sequence

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Nomenclature

A_t	total area of heat removal surface of heat sink, m^2
C_P	specific heat of air, J/kg K
D_i	inner diameter of air-flow channel, m
D_o	outer diameter of air-flow channel, m
DOF	number of degrees of freedom
F_{factor}	F-ratio of the factor
h_{av}	average convective heat transfer coefficient, W/m^2 K
L	number of factor's level
$\dot{m}$	mass-flow rate, kg/s
n	number of experiments
ΔP	pressure drop, Pa
Q	heat transfer rate, J/s
R_{th}	thermal resistance, K/W
SS_{factor}	sum of factorial squared deviations
SS_{total}	sum of total squared deviations
T	steady-state temperature, K
ΔT_m	log mean temperature difference, K
T_{max}	highest temperature of tested heat sink base, K
V_{factor}	variance of the factor
V_{in}	mean speed of the air in the air-flow channel, m/s
W	heat dissipation produced by the heating unit, W (J/s)
$X_i(k)$, $i \neq 0$	objective sequence
$X_0(k)$	reference sequence

Greek symbols

β	distinguishing coefficient
η_0	grey–fuzzy reasoning grade
μ	corresponding membership functions
ξ	grey relational coefficient

Subscripts

in	inlet
out	outlet
s	surface

(a collection of measurements or actual experimental results) and a reference sequence (desired experimental results or target values) in the grey system. A grey relational grade is obtained from the average of the grey relational coefficient so as to provide an optimal machining parameter in which the manufacture simultaneously requests multiple performance characteristics [10,11].

The theory of fuzzy logic originated by Zadeh [12] is an effective mathematical module for resolving problems that contain an uncertain and considerable amount of information. The fuzzy logic analysis, including the max–min fuzzy inference and centroid defuzzification method [13], adopts the fuzziness of human concepts to deal with multiple performance characteristics. Therefore, the fuzzy logic can also be applied to establish the optimal set of control parameters with the considerations of multiple performance characteristics. According to the fuzzy logic, a grey–fuzzy reasoning grade has been developed to handle the grey relational coefficient of multiple performance characteristics in this study. The grey-based fuzzy algorithm, integrating the grey relational analysis and fuzzy logic, has successfully been used in the high-tech engineering. Lin *et al.* [14] applied the Taguchi method with fuzzy logic for optimizing the machining process with multiple performance characteristics in the electrical discharge machining process. Wang *et al.* [15] proposed a fuzzy–grey prediction procedure based on the symmetric fuzzy number, linear planning theory and grey set theory for predicting the extent of turning force uncertainty quantitatively. Lin and Lin [16] made use of the grey–fuzzy logic based on orthogonal array for optimizing the electrical discharge machining process with multiple process responses. Albert *et al.* [17] developed an integrated grey–fuzzy-based electricity management system for providing an efficient means to control and manage the use of electricity for enterprises. Karmakar and Mujumdar [18] developed a grey–fuzzy optimization model for water quality management of a river system. Grum and Slabe [19–21] made use of factorial design and response surface methodology to the determination of the optimum heat-treatment conditions of the surfaced layers made from different Ni–Co–Mo alloys.

The main objective of this study is to employ the grey relational analysis and the fuzzy logic for establishing the settings of optimal designing parameters for the heat sink with Parallel-Plain Fin (PPF) on the multiple thermal performance characteristics. The heat sink with PPF is widely applied in the cooling enhancement of current electronic equipment. Because of the advantage of easy machining, simple structure and lower cost, various forms of heat sinks with PPF are manufactured and supplied to electronic industry in a large quantity. In the present case study, the designing parameters of the heat sink with PPF include the height of fin, the width of gap between fins, the width of slot, the number of slot and the air speed. The average convective heat transfer coefficient, the thermal resistance and the pressure drop are considered to be multiple thermal performance characteristics and explored in the experiment. In addition, the response table, response graph and analysis of variance (ANOVA) [22–24] are used to find the optimal settings and the influence of designing parameters on the multiple thermal performance characteristics.

2. The integrated algorithm of grey relational analysis and fuzzy logic

2.1. Grey relational analysis

The grey represents the primitive data with poor, incomplete and uncertain information, and the incomplete relationship of the information among these data is called the grey relation. Grey relational analysis uses the quantitative analysis to describe the degree of relationship between an objective sequence (a collection of measurements or experimental results) and a reference sequence (target value) in the grey system. The measurement of relationship between two sequences mentioned above can be expressed as the grey relational coefficient [8,9]. In the grey system, the set of sequence X_i is expressed as follows:

$$X_i = [X_i(1), X_i(2), \ldots, X_i(k)], \quad i \in I, \ k \in N, \tag{1}$$

where $X_i(k)$, $i \neq 0$, is the objective sequence and $X_0(k)$ is the reference sequence. In the procedure of grey relational analysis, the raw sequences are first normalized in the range between 0 and 1 due to the different measurement units and scales, and this process is called the grey relational generation. The normalized results reveal the situation of better performance in the grey relational analysis. The larger value of normalized results can express the better performance, and the best-normalized results will be equal to one. The normalized type depends on the characteristics of raw sequence including the larger-the-better, smaller-the-better and nominal-the-better characteristics. Consequently, the type of normalized results can be expressed in the following:

for the larger-the-better characteristic

$$X_i^*(k) = \frac{X_i(k) - \min\limits_{\forall k} X_i(k)}{\max\limits_{\forall k} X_i(k) - \min\limits_{\forall k} X_i(k)}, \tag{2}$$

for the smaller-the-better characteristic

$$X_i^*(k) = \frac{\max\limits_{\forall k} X_i(k) - X_i(k)}{\max\limits_{\forall k} X_i(k) - \min\limits_{\forall k} X_i(k)}, \tag{3}$$

and for the nominal-the-better characteristic

$$X_i^*(k) = 1 - \frac{|X_i(k) - X_{ob}(k)|}{\max\limits_{\forall k} X_i(k) - \min\limits_{\forall k} X_i(k)}, \tag{4}$$

where $\max\limits_{\forall k} X_i(k)$ and $\min\limits_{\forall k} X_i(k)$ are the largest and smallest values of $X_i(k)$, respectively, and $X_{ob}(k)$ is the target of $X_i(k)$. The grey relational coefficient $\xi_i(k)$ for $X_i(k)$ to $X_0(k)$ is calculated as follows:

$$\xi_i(k) = \frac{\min\limits_{i\in I}\ \min\limits_{k}\left|X_0^*(k) - X_i^*(k)\right| + \beta \max\limits_{i\in I}\ \max\limits_{k}\left|X_0^*(k) - X_i^*(k)\right|}{|X_0^*(k) - X_i^*(k)| + \beta \max\limits_{i\in I}\ \max\limits_{k}\left|X_0^*(k) - X_i^*(k)\right|}, \tag{5}$$

where $|X_0^*(k) - X_i^*(k)|$ is the absolute difference of two comparative sequences, $\min\limits_{i\in I}\ \min\limits_{k} |X_0^*(k) - X_i^*(k)|$ and $\max\limits_{i\in I}\ \max\limits_{k} |X_0^*(k) - X_i^*(k)|$ are the minimum and maximum values of $|X_0^*(k) - X_i^*(k)|$ respectively, and β is the distinguishing coefficient whose value is adjusted according to the systematic actual status and defined in the range between 0 and 1. Here the value of β will be set to 0.5, the quantity applied in most situations. The grey relational coefficient is applied to display the relationship between the optimal (best = 1) and actual normalized results. A higher grey relational coefficient $\xi_i(k)$ represents that the corresponding experimental result is closer to the optimal (best) normalized value for the single response.

2.2. Fuzzy logics

In fact, the grey relational coefficient $\xi_i(k)$ for each corresponding sequence contains some degree of uncertainty and vagueness in the definition of response characteristics, including the larger-the-better, smaller-the-better and nominal-the-better characteristics. A fuzzy reasoning of the multi-response characteristics based on the fuzzy logics is proposed to obtain the optimal factors/levels combination in a multi-responses problem. In the procedure of fuzzy reasoning, the fuzzifier initially uses membership functions to fuzzify the grey relational coefficient $\xi_i(k)$ of each sequence. A membership function (MF) is a curve to determine how each input value is mapped to a membership value (or degree of membership) between 0 and 1. The inference engine performs a fuzzy interface so as to generate a fuzzy value according to the membership function and fuzzy rules. The aggregation of a fuzzy set must be defuzzified by the centroid defuzzification method [12,13]. Hence, the defuzzifier can convert the fuzzy value into the non-fuzzy value called the grey–fuzzy reasoning grade in this procedure.

In this study, the membership function adopts the trapezoidal membership function that has a flat top and a truncated triangle curve. In fuzzy logic, these if–then rule statements are used to formulate the conditional statements that have the multi-response grey relational coefficients ξ_n and one multi-response output η, that is,

$$\begin{aligned}\text{Rule 1}:\ & \text{if } \xi_1 \text{ is } A_{11} \text{ and } \xi_2 \text{ is } A_{12}\ldots \text{ and } \xi_n \text{ is } A_{1n}\\ & \text{then } \eta \text{ is } D_1 \text{ else},\end{aligned} \tag{6a}$$

$$\begin{aligned}\text{Rule 2}:\ & \text{if } \xi_1 \text{ is } A_{21} \text{ and } \xi_2 \text{ is } A_{22}\ldots \text{ and } \xi_n \text{ is } A_{2n}\\ & \text{then } \eta \text{ is } D_2 \text{ else},\end{aligned} \tag{6b}$$

$$\begin{aligned}\text{Rule } i:\ & \text{if } \xi_1 \text{ is } A_{i1} \text{ and } \xi_2 \text{ is } A_{i2}\ldots \text{ and } \xi_n \text{ is } A_{in}\\ & \text{then } \eta \text{ is } D_i \text{ else},\end{aligned} \tag{6c}$$

$$\begin{aligned}\text{Rule } n:\ & \text{if } \xi_1 \text{ is } A_{n2} \text{ and } \xi_2 \text{ is } A_{n2}\ldots \text{ and } \xi_n \text{ is } A_{nn}\\ & \text{then } \eta \text{ is } D_n\end{aligned} \tag{6d}$$

where A_{i1}, A_{i2}, …, A_{in} and D_i are the fuzzy subsets defined by the corresponding membership functions, i.e., $\mu_{A_{i1}}, \mu_{A_{i2}}, \ldots, \mu_{A_{in}}$, and μ_{D_i}. The fuzzy multi-response output η is provided from above those rules by employing the max–min interface operation. Inference results in a fuzzy set with membership function for the multi-response output η can be expressed as follows:

$$\begin{aligned}\mu D_0(\eta) = {} & \left(\mu_{A_{11}}(\xi_1)\wedge\mu_{A_{12}}(\xi_2)\wedge\mu_{A_{13}}(\xi_3)\wedge\cdots\mu_{A_{1n}}(\xi_n)\wedge\mu_{D_1}(\eta)\right)\cdots\vee\\ & \left(\mu_{A_{n1}}(\xi_1)\wedge\mu_{A_{n2}}(\xi_2)\wedge\mu_{A_{n3}}(\xi_3)\wedge\cdots\mu_{A_{nn}}(\xi_n)\wedge\mu_{D_n}(\eta)\right),\end{aligned} \tag{7}$$

where $\wedge$ and $\vee$ are the minimum and maximum operations, respectively.

Finally, the fuzzy multi-response output $\mu D_0(\eta)$ must be transferred to a non-fuzzy value η_0 by the calculation of centroid defuzzification method, that is,

$$\eta_0 = \frac{\sum \eta\mu D_0(\eta)}{\sum \mu D_0(\eta)}. \tag{8}$$

This non-fuzzy value η_0 is called grey–fuzzy reasoning grade. The grey–fuzzy reasoning grade η_0 can handle the optimization of complicated multiple machining responses. Using the value of grey–fuzzy reasoning grade η_0, the relational degree between main factor and other factors is calculated for each response characteristic. Hence a higher value of grey–fuzzy reasoning grade η_0 indicates that experimental result is close to the ideally normalized value.

2.3. The proposed optimal procedure

The integrated algorithm combines the grey relational analysis with fuzzy logic in order to determine the designing parameters with optimal performance characteristics, and its result is illustrated in Fig. 1. In the procedure of grey–fuzzy logic analysis, the optimization of the complicated multiple performance characteristics can be converted into the optimization of a single grey–fuzzy reasoning grade. These proposed algorithms, including seven steps, are summarized as follows [14–18,22–24]:

Step (1) Adopt an appropriate orthogonal array to plan the experimental design and determine the level of designing parameters.
Step (2) Define the performance evaluation and normalize the experimental results $X_i(k)$, $i \neq 0$, by using formula (2)–(4).
Step (3) Use formula (5) for each performance characteristic to calculate the grey relational coefficient $\xi_i(k)$.

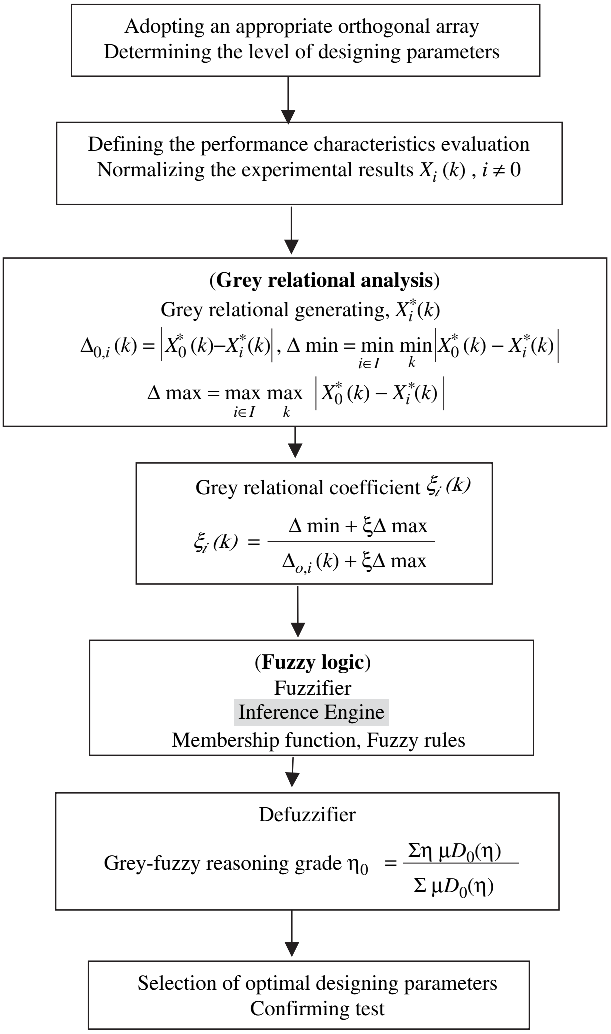

Fig. 1. Structure of the grey–fuzzy integrated algorithm for the optimal designing parameters.

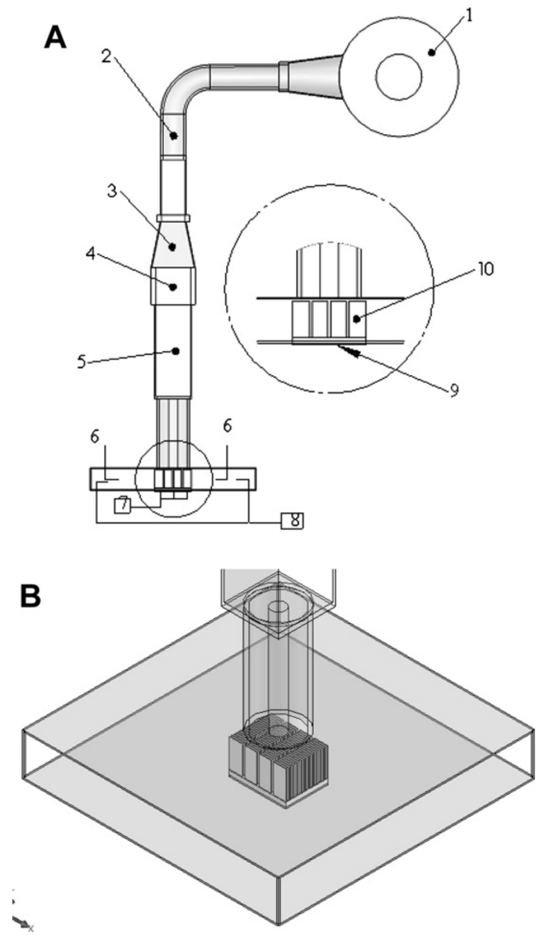

Fig. 2. A schematic display of the experimental setup. 1. Blower, 2. preheat, 3. chamber, 4. adjustable contraction zone, 5. honeycomb, 6. pitot tube, 7. hot-wire sensor, 8. thermocouple sensor, 9. heating unit (DC power supply), 10. heat sink.

Step (4) Establish the trapezoidal membership function and fuzzy rule to fuzzify the grey relational coefficient $\xi_i(k)$ of each response.
Step (5) Calculate the fuzzy multi-responses output $\mu D_0(\eta)$ by the max–min interface operation and use formula (8) to transfer the $\mu D_0(\eta)$ into a grey–fuzzy reasoning grade η_0.
Step (6) Perform the response table and response graph in order to select the optimal level setting of designing parameters.
Step (7) Confirm tests and verify the optimal setting of designing parameters.

3. Test setup and procedure

3.1. Experimental apparatus

A schematic diagram of the test setup used in the experiments is shown in Fig. 2(A). The experimental apparatus consisted of air blower, pre-heater, adjustable contraction zone, honeycomb, air-flow channel, test section, and measurement facilities. Air was supplied by a centrifugal blower with the variable-speed drive and then passed through an insulated chamber. The pre-heater controlled the air temperature at inlet of the air-flow channel. After the chamber, the apparatus of adjustable contraction zone and honeycomb were utilized to adjust the status of air-flow to laminar flow. The air-flow channel was made of a cannular cylinder and mounted in a vertical position on top of the test section. This structure was designed to simulate the electric micro-fan (an external wing diameter of 65 mm and a motor diameter of 25 mm) installed over the heat sink. The static pressure in air-flow channel was measured by using a static-pressure tapping located within the middle of this channel. The pressure tapping was connected to an inclined manometer. The test section in Fig. 2(B) was constructed by a hollow rectangular block ($720 \times 720 \times 150$ mm) made up of upper and bottom plates. It was made of Plexiglas plate of 20 mm thickness. Air can exhaust through the horizontal vicinity of test section. The outside of the air-flow channel and test section were insulated with a layer of glass-wool insulation. The outlet temperature of the air stream located in the horizontal vicinity of test section was measured with a thermocouple and data acquisition system. The static-pressure tapping measured the static pressure of air stream exhausted from the horizontal vicinity of test section. The heating unit was located in the middle of bottom plate which consists of the electric heater, voltage transformer, a firebrick of 25 mm thickness and the thermal insulation. The electric heater and voltage transformer were used to control the heat flux along the bottom of base plate. At first the heat generated by heating unit was conducted through the heat sink and then diffused to the environment by means of forced convection.

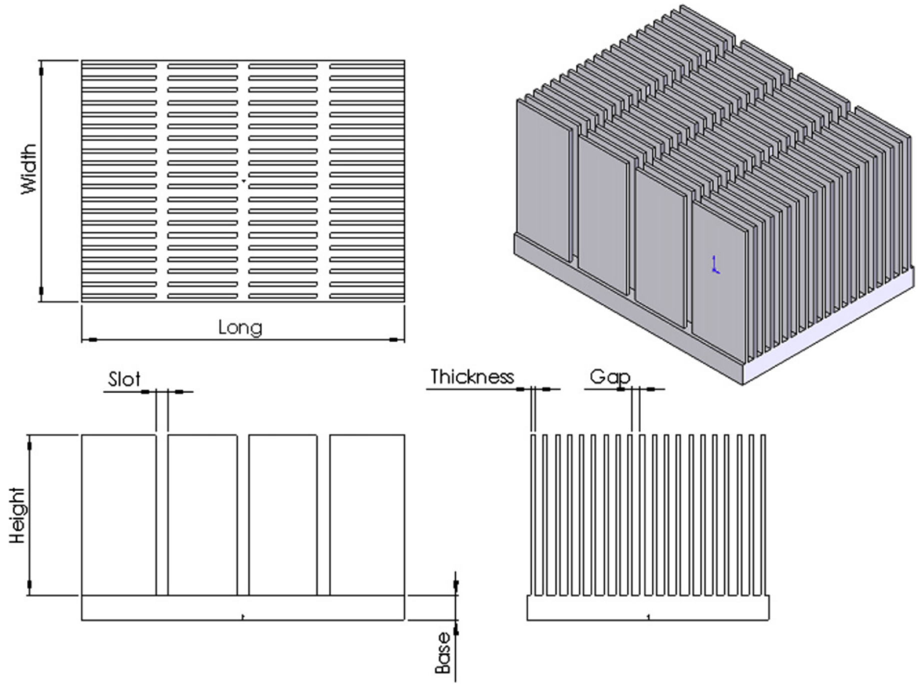

Fig. 3. Schematic diagrams of the commercial extruded heat sink with PPF.

The schema of the commercial extruded heat sink with PPF in this case study is shown in Fig. 3. The material of the heat sink is the aluminum alloy 6063-T5 (thermal conductivity 209 W/m K). The manufacturing method of the heat sink is first to extrude the aluminum alloy 6063-T5 through a die and mill it by using CNC milling machine. The heat sink has the $80 \times 60 \times 5$ mm base size which is suitably mounted on the CPU. The tested heat sink was mounted on the heating unit with two small tapped holes. The base of the heat sink was heated uniformly with a heat load of 40 W. The steady-state temperature of the heat sink base was measured by using the twenty-five gauge (0.12 mm diameter wire) copper-constantan thermocouple installed on the base plate. The data of temperature was instantaneously scanned by using a data acquisition and calibrated within ± 0.01 K.

3.2. Performance evaluation of heat sink module

The thermal performance of heat sink module is evaluated by the heat transfer rate and the capacity of an axial-flow cooling fan installed over the tested heat sink. For a given operating condition of cooling fan, increasing the heat transfer rate is a principal cause of high thermal performance (or cooling efficiency). Here we adopt the thermal resistance R_{th} and the average convective heat transfer coefficient h_{av} to evaluate the thermal performance. The steady-state rate of heat transfer to the flowing air through the tested heat sink can be calculated by

Table 1
Design scheme of designing parameters and their levels.

Symbol	Designing parameters	Unit	Level			
			1	2	3	4
A	Air speed	m/s	1.5*	2	2.5	3
B	Height of fin	mm	35	40*	45	40
C	Width of gap between fins	mm	1.00	1.25*	1.50	1.75
D	Width of slot	mm	1.5	3*	4.5	6
E	Number of slot		1	3*	5	7

Remark (*) indicates the original designing value, total number of fin is restricted to be 20, and thickness of fin is designed to be 1 mm.

$$Q = \dot{m} C_P (T_{out} - T_{in}), \tag{9}$$

where C_P is the specific heat of air. The mass-flow rate $\dot{m}$, based on mean speed V_{in} of the air in the air-flow channel, is defined as

$$\dot{m} = \rho V_{in} \left(D_o^2 - D_i^2 \right) \pi / 4. \tag{10}$$

The heat transfer rate across the tested heat sink can also be expressed as,

$$Q = h_{av} A_t \Delta T_m, \tag{11}$$

where ΔT_m is the log mean temperature difference that estimated

$$\Delta T_m = (\Delta T_i - \Delta T_o) / \ln(\Delta T_i / \Delta T_o), \tag{12}$$

where $\Delta T_i = T_S - T_{in}$ and $\Delta T_o = T_S - T_{out}$. A_t is the total area of heat removal surface of test heat sink. Hence the average convective heat transfer coefficient h_{av} can be obtained as following

Table 2
Experimental layout using the $L_{16}(4^5)$ orthogonal array.

Exp No.	Designing parameter					h_{av} (W/m^2 K)	R_{th} (K/W)	ΔP (Pa)
	A	B	C	D	E			
1	1	1	1	1	1	81.729	0.344	23.94
2	1	2	2	2	2	82.324	0.341	24.65
3	1	3	3	3	3	81.697	0.332	25.03
4	1	4	4	4	4	83.591	0.336	23.22
5	2	1	2	3	4	83.562	0.351	24.56
6	2	2	1	4	3	83.509	0.350	22.36
7	2	3	4	1	2	83.982	0.350	23.53
8	2	4	3	2	1	84.457	0.332	23.78
9	3	1	3	4	2	84.168	0.339	22.02
10	3	2	4	3	1	85.738	0.340	22.41
11	3	3	1	2	4	83.669	0.344	23.33
12	3	4	2	1	3	84.420	0.349	22.29
13	4	1	4	2	3	82.999	0.346	24.62
14	4	2	3	1	4	82.739	0.349	21.73
15	4	3	2	4	1	82.253	0.350	25.99
16	4	4	1	3	2	83.016	0.344	21.70

Table 3
The grey coefficient $\xi_i(k)$ for h_{av}, R_{th} and ΔP.

No.	$\xi_1(k)$	$\xi_2(k)$	$\xi_3(k)$
1	0.3471	0.4555	0.4998
2	0.3838	0.5208	0.4321
3	0.3453	1.0000	0.4030
4	0.4968	0.6942	0.5968
5	0.4935	0.3383	0.4396
6	0.4875	0.3548	0.7738
7	0.5470	0.3564	0.5500
8	0.6239	0.9869	0.5183
9	0.5748	0.5966	0.8784
10	1.0000	0.5603	0.7614
11	0.5061	0.4555	0.5786
12	0.6172	0.3658	0.7932
13	0.4365	0.4064	0.4344
14	0.4145	0.3664	0.9969
15	0.3790	0.3498	0.3443
16	0.4380	0.4537	1.0000

$$h_{av} = Q/(A_t \Delta T_m). \tag{13}$$

In this study, the value of h_{av} is regarded as the performance characteristic of heat removal capacity of the PPF heat sink module. The other performance characteristic is the thermal resistance, R_{th}, which calculated as

$$R_{th} = (T_{max} - T_{in})/W, \tag{14}$$

where T_{max} is the highest temperature of tested heat sink base and W is the heat dissipation produced by the heating unit. Furthermore, the pressure drop ΔP through the tested heat sink is also regarded as the thermal performance characteristic because it affects the amount of airflow through the test section or the status of bypass effect. The function of increasing pressure drop promotes the pumping power of axial-flow cooling fan installed over the tested heat sink. The difference of pressure in the air-flow channel and horizontal vicinity of test section acquires the quantity of pressure drop ΔP. For a fan-driven heat sink, the high thermal performance (or cooling efficiency) can be obviously achieved by maximizing the average convective heat transfer coefficient h_{av}, and by minimizing both the thermal resistance R_{th} and the pressure drop ΔP in the case of the fixed operating condition. The h_{av} is regarded as the larger-the-better characteristic, and the R_{th} and ΔP are regarded as the smaller-the-better characteristics, and they influence each relatively.

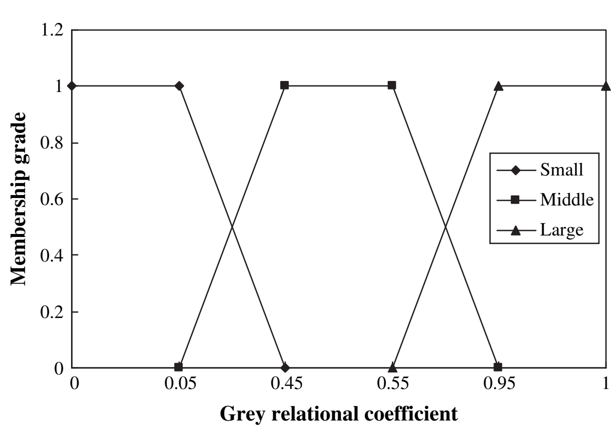

Fig. 4. The trapezoidal membership functions for the multiple thermal performance characteristics.

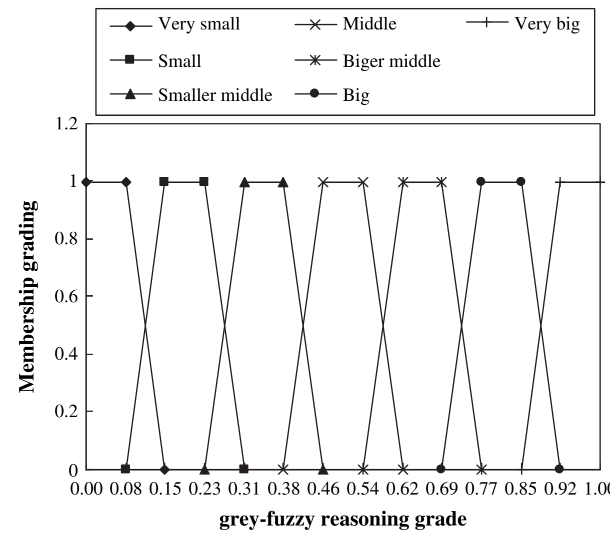

Fig. 5. The seven fuzzy subsets of multiple performance output η.

3.3. Design of experiment

The settings of experimental plan affecting the heat transfer capacity of the tested heat sink are determined by using the Taguchi experimental design method. The benefit of Taguchi method with the robust design is to simplify a great quantity of factor experimentation based on the design of experiments (DOE) [22–24]. This factorial experimental design is called orthogonal array and becomes a more effective method for practicing engineers and scientists.

The designing parameters of the heat sink with PPF in the present study include the height of fin, the width of gap between fins, the width of slot, the number of slot and the air speed. As shown in Table 1, the designing parameters with four levels were selected to investigate the influence of heat removal capability. In this study, the factors of designing parameters and the factor levels were identified according to the spacing of CPU boards mounted on the motherboard. In Table 1, the remark (*) indicates the original designing value, the number of total fins is restricted to be 20, and the thickness of fin is designed to be 1 mm. The $L_{16}(4^5)$ orthogonal arrays table was chosen for performing the experimental design. The interaction between the designing parameters was neglected in the present study. Only 16 experiments in Table 2 were applied to conduct this experimental plan according to the $L_{16}(4^5)$ orthogonal arrays. Under the same conditions, each combination in the

Table 4
The grey–fuzzy reasoning grade η_0.

No.	Grey–fuzzy reasoning grade η_0	Order
1	0.4691	13
2	0.4806	12
3	0.6178	8
4	0.6309	5
5	0.4588	15
6	0.5737	9
7	0.5195	11
8	0.7447	2
9	0.7183	3
10	0.8089	1
11	0.5484	10
12	0.6271	7
13	0.4608	14
14	0.6276	6
15	0.3927	16
16	0.6656	4

Table 5
Response table for the grey–fuzzy reasoning grade η_0.

Symbol	Designing parameter	Grey–fuzzy reasoning grade				
		Level 1	Level 2	Level 3	Level 4	Max–min
A	Air speed	0.5367	0.5496	0.5742	0.6757	0.1390
B	Height of fin	0.5196	0.5268	0.6227	0.6671	0.1475
C	Width of gap between fins	0.4898	0.5642	0.6771	0.6050	0.1873
D	Width of slot	0.5586	0.5608	0.6378	0.5789	0.0791
E	Number of slot	0.6039	0.5960	0.5698	0.5665	0.0374
Total mean value of the grey–fuzzy reasoning grade $\eta_0 = 0.5840$						

Table 6
Results of the analysis of variance.

Symbol	Designing parameter	DOF	Sum of squares	Variance	F	Contribution (%)
A	Air speed	3	0.0477	0.0159	2.9088	23.25
B	Height of fin	3	0.0633	0.0211	3.8607	30.86
C	Width of gap between fins	3	0.0735	0.0245	4.4838	35.85
D	Width of slot	3	0.0164	0.0055	1.0000	7.99
E	Number of slot	3	0.0042	0.0014	0.2554	2.04
	Error		0.0000			
	Total	15	0.1210			100.00

experiments was repeated three times at different times to acquire a more accurate result in this process.

3.4. Calculating the grey relational coefficient

The results from the thermal performance evaluation of the heat sink in each experimental plan were also tabulated in Table 2. According to the procedure of grey relational analysis, the experimental results of h_{av}, R_{th} and ΔP were first normalized by using formula (2)–(4). The grey relational coefficient $\xi_i(k)$ for each machining response was calculated by formula (5) and listed in Table 3. The experimental Nos. 10, 3 and 16 in Table 3 have the best performance characteristics for the h_{av}, R_{th} and ΔP, respectively.

3.5. Calculating the grey–fuzzy reasoning grade

Fig. 4 shows that three fuzzy subsets are assigned in the grey relational coefficient of h_{av}, R_{th}, and ΔP by using the trapezoidal membership function. In the fuzzy logic, these if–then rule statements are used to formulate the conditional statements that have the three grey relational coefficients, ξ_1, ξ_2, and ξ_3, and one multi-performance output, η, that is,

$$\text{Rule 1}:\ \text{if } \xi_1 \text{ is } A_{11} \text{ and } \xi_2 \text{ is } A_{12} \text{ and } \xi_3 \text{ is } A_{13} \text{ then } \eta \text{ is } D_1 \text{ else,} \tag{15a}$$

$$\text{Rule 2}:\ \text{if } \xi_1 \text{ is } A_{21} \text{ and } \xi_2 \text{ is } A_{22} \text{ and } \xi_3 \text{ is } A_{23} \text{ then } \eta \text{ is } D_2 \text{ else,} \tag{15b}$$

$$\text{Rule 3}:\ \text{if } \xi_1 \text{ is } A_{31} \text{ and } \xi_2 \text{ is } A_{32} \text{ and } \xi_3 \text{ is } A_{33} \text{ then } \eta \text{ is } D_3. \tag{15c}$$

The seven fuzzy subsets can be applied to the multi-performance output η, as shown in Fig 5. Hence the 21 fuzzy rules based on the occurrence are directly obtained and the larger grey relational coefficient is the better performance characteristic. The inference results in a fuzzy set with membership function for the multi-performance output η can be expressed as

$$\mu D_0(\eta) = \big(\mu_{A_{11}}(\xi_1)\wedge\mu_{A_{12}}(\xi_2)\wedge\mu_{A_{13}}(\xi_3)\wedge\mu_{D_1}(\eta)\big)\vee \big(\mu_{A_{21}}(\xi_1)\wedge\mu_{A_{22}}(\xi_2)\wedge\mu_{A_{23}}(\xi_3)\wedge\mu_{D_2}(\eta)\big)\vee\big(\mu_{A_{31}}(\xi_1)\wedge \mu_{A_{32}}(\xi_2)\wedge\mu_{A_{33}}(\xi_3)\wedge\mu_{D_2}(\eta)\big), \tag{16}$$

where $\wedge$ and $\vee$ are the minimum and maximum operations, respectively. The fuzzy multi-performance output $\mu D_0(\eta)$ has been transferred to a grey–fuzzy reasoning grade η_0 according to formula (8), and the results are tabulated in Table 4. In this study, the experiment No. 10 has the best multiple (total) heat removal characteristics among the 16 experiments in Table 4.

4. Results and discussions

The response table and response graph are obtained from the average value of the grey–fuzzy reasoning grade η_0 for each level of the designing parameters in order to find the optimal levels of four designing parameters. Since the experimental design is orthogonal, it is then possible to separate out the grey–fuzzy reasoning grade η_0 of each designing parameter at different levels in Table 5. For example, the mean of the grey–fuzzy reasoning grade η_0 for the air speed at levels 1, 2, 3 and 4 can be calculated by averaging the grey–fuzzy reasoning grade η_0 for experiments 1–4, 5–8, 9–12 and 13–16, respectively (Table 2). The mean of the grey–fuzzy reasoning grade η_0 for each level of the other machining parameters can be computed in a similar manner. The optimal setting of designing parameter is to select the level with the higher value of grey–fuzzy reasoning grade η_0. Since the grey–fuzzy reasoning grade η_0 represents the level of relationship between the reference sequence and the objective sequence, a greater value of grey–fuzzy reasoning grade η_0 reveals that the objective sequence has a stronger correlation to the reference sequence. Therefore, the value of grey–fuzzy reasoning grade η_0 for each level is the greater the better in the response table. From Table 5 and Fig. 6, the level

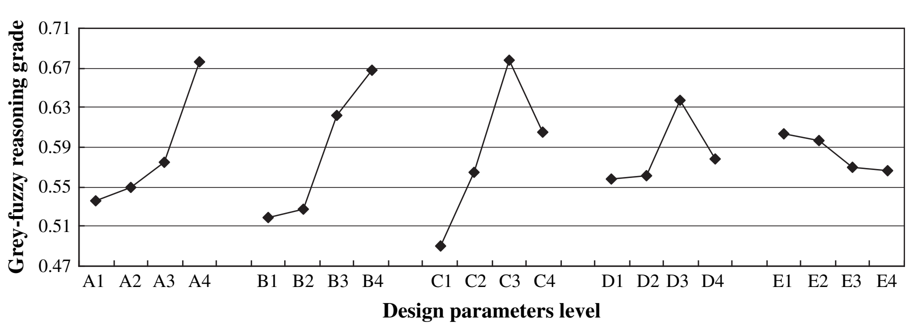

Fig. 6. The response graph for each level of the designing parameters.

Table 7
The comparing results of the initial and optimal designing parameters.

	Initial condition	Optimal condition		Deviation %
		Prediction	Experiment	
Designing parameter level	A1, B2, C2, D2, E2	A4, B4, C3, D3, E1	A4, B4, C3, D3, E1	
h_{av} (W/m^2 K)	82.324	96.241	95.372	0.91
R_{th} (K/W)	0.341	0.264	0.262	0.76
ΔP (Pa)	24.65	20.43	20.64	−1.01
Grey–fuzzy reasoning grade η_0	0.486	0.8721	0.8632	1.03
Improvement of the grey–fuzzy reasoning grade		0.3815	0.3772	

constitution of optimal designing parameters are A4, B4, C3, D3, and E1 for maximizing the value of h_{av}, and for minimizing the value of R_{th} and ΔP, simultaneously. A steeper slope of response graph indicates more influential designing parameter on the performance characteristic in Fig. 6. Results also show that the first three designing parameters namely (C) the width of gap between fins, (B) height of fin and (A) air speed have greater value of steep slope and will have significant influence for affecting the multiple (total) heat removal characteristics.

In this case study, the analysis of variance (ANOVA) is carried out to examine the influence of designing parameters on the performance characteristics. The results of ANOVA determine the percent contribution of designing parameters in order to obtain the influential degree of designing parameters on the total performance characteristics. In general, the percent contribution of designing parameters can be calculated by the following equations.

$$SS_{\text{total}} = \sum_{i=1}^{n} \eta_{0i}^2 - n(\eta_{0m})^2, \quad SS_{\text{factor}} = \frac{n}{L}\sum_{i=1}^{L} (\eta_{0i} - \eta_{0m})^2, \tag{17}$$

$$SS_{\text{error}} = SS_{\text{total}} - \sum SS_{\text{factor}}, \tag{18}$$

$$\text{DOF}_{\text{total}} = n - 1, \quad \text{DOF}_{\text{factor}} = L - 1, \tag{19}$$

$$V_{\text{factor}} = \frac{SS_{\text{factor}}}{\text{DOF}}, \quad F_{\text{factor}} = \frac{V_{\text{factor}}}{V_{\text{error}}}, \tag{20}$$

where SS_{total} is the sum of total squared deviations, SS_{factor} is the sum of factorial squared deviations, n is the number of experiments, DOF is the number of degrees of freedom, L is the number of factor's level, V_{factor} is the variance of the factor, F_{factor} is the F-ratio of the factor, η_{0i} is the grey–fuzzy reasoning grade η_0 of each experiment, and η_{0m} is the total mean of the grey–fuzzy reasoning grade. Statistically, Fisher's F test [24] provides a decision at some confidence level whether these parameters are significantly effective on the performance characteristics. A large value of F-ratio indicates that the change of the designing parameter makes a significant effect upon the performance characteristic.

From the results of ANOVA in Table 6, the contribution percentage of first three significant machining parameters namely (C) the width of gap between fins, (B) the height of fin and (A) the air speed are 35.85, 30.86 and 23.25%, respectively. From the F-test analysis, those parameters have been regarded to be the significant factor again. Therefore the above three designing parameters are noticeably variable factors as the resources of increasing capability of heat removal. It is also seen from Table 6 that (D) the width of slot and (E) the number of slot affect the total performance characteristics by 7.99 and 2.04%, respectively. Those designing parameters are regarded as unnoticeably variable factors so as to reduce manufacturing costs.

Since the optimal level of the designing parameters is selected, the confirmation test is processed to verify the improvement of total performance characteristics. The comparisons between these predicted optimal designs and those measured by the experimental data can be easily comprehended and are listed in Table 7. Under designing constraints, the initial designing parameters were designated as A1, B2, C2, D2, and E2, which is the experiment No. 2 in Table 3. The accuracies of total performance characteristics have been verified by comparing the predicted response values to the actual experimental data. The maximum deviations of h_{av}, R_{th}, ΔP are 0.91, 0.76 and −1.01%, respectively. The maximum deviation of η_0 predicted by the present algorithm from the experimental data is less than 1.03%. As shown in Table 7, the confirmation test results of optimal designing parameters indicate that the value of h_{av} is improved from 82.324 to 95.372 (W/m^2 K), the value of R_{th} is reduced from 0.341 to 0.262 (K/W) and the value of ΔP is decreased from 24.56 to 20.34 (Pa). The grey–fuzzy reasoning grade η_0 will increase to 0.8632, which is larger than any other grey–fuzzy reasoning grade η_0 in Table 4. Consequently, these results prove that the procedure of grey–fuzzy logic with orthogonal arrays can be successfully applied for determining the multiple heat removal characteristics in the heat sink model with PPF, with only a limited number of experiments and the shorter time is needed to achieve the optimal design parameters setting.

5. Conclusions

In this study, a fast and effective algorithm integrating the grey relational analysis and the fuzzy logic is applied to find the optimal setting of design parameters for a heat sink with Parallel-Plain Fin (PPF) on the multiple heat removal characteristics. As already stated, the conclusions drawn are as follows:

(1) The proposed algorithm acquires a grey–fuzzy reasoning grade so as to evaluate the multiple performance characteristics. It is more effective to make the determination of optimal setting and can greatly simplify the optimization procedure for the complicated multiple performance characteristics.
(2) The level constitution of optimal designing parameters are A4, B4, C3, D3, and E1 for maximizing the value of average convective heat transfer coefficient, and for minimizing the value of both thermal resistance and pressure drop, simultaneously.
(3) The width of gap between fins, the height of fin and the air speed are the most significant machining parameters with contribution percentage of 35.85, 30.86 and 23.25%, respectively, for the heat sink on the multiple (total) heat removal characteristics.
(4) From the results of confirmation test, the multiple performance characteristics, including the average convective heat transfer coefficient, the thermal resistance and the pressure drop, have a considerable improvement for the heat sink by using the algorithm proposed in this study. Under designing constraints, the comparisons between these predicted optimal designs and those measured by the experimental data were made with a satisfactory agreement.

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